EMITTANCE EVALUATION ALGORITHMS

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Abstract
Measurement of beam emittance remains an important part of accelerator diagnostics. The present tendency for charged particles sources and accelerators is providing beams of improved quality or/and higher intensity. Stability, size, energy spread etc. are the subject of new researches. The community demands an adequate set of measurement devices to get the possibility to control and optimize beam parameters. This stimulates a generation of new ideas in the measurement environment as well as new ways to evaluate the measured data. The measurement costs also increase due to higher resolution or higher energy or shorter time of investigated phenomena. That fact inspires the interest to high quality algorithms allowing better results with less data.

1. DATA ACQUISITION

Data acquisition methods
All classical methods of emittance measurements use one-dimensional detectors. Usually it is a row of charge or current-sensitive elements which is able to reproduce a spatial distribution of incident beam. Consequently, the original data dimensions are reduced to meet the detector capability. Figure 1.1 shows the particle separation by coordinate criteria. This corresponds to the mostly used method in emittance measurements.

Use of high resolution detectors allows an advanced modification of method such as pepper-pot with single shot measurement possibility. Another way to eliminate extra dimensions is shown on figure 1.2. An adjustable collimator produces a set of particles with fixed inclination. The collected data allows a reproduction of original phase space distribution when a whole range of angles is covered. This method requires successive operation and could not be used in single-shot measurements. But it produces a sufficient amount of data for investigation of non-trivial full-dimensional distributions.

In contrast a beam with canonical charge distribution could be investigated in much simpler way. Sometimes measurements could be made even on non-destructive manner using beam width measurements in series of locations or by varying parameters of beam optics elements as it is shown on figure 1.3.

Figure 1.1. Particle separation by coordinate.

Figure 1.2. Particle separation by angle.

Figure 1.3. Beam matrix S could be reproduced by measurements of beam width S’11 evolution as a function of quadrupole strength k.

Data acquisition software
The software for the emittance data acquisition must provide a reliable and convenient way for planning, parameterization and executing of measurements. Depending on the application the software gives an access to the parameter set including for instance a slit-grid.
movement span, resolution or integration time for slit-grid devices or exposure time for pepper-pot systems. The software for stationary devices often implemented as a "one click" process, which is reasonable for measurements of small variations of the same beams when no extensive system tuning is required. Figure 1.4 shows the example of the interface of the software which belongs to GSI’s slit-grid MobiEmi measurement system. Actually it works with several physical devices and was used for commissioning of the various accelerators with wide range of beam parameters.

Figure 1.4. Layout of the slit-grid control software. Different windows provide an extended control over device parameters. An off-beam test mode, constrained and multi-pass measurements are the options of the acquisition software.

Certain care should be taken on the data pre-processing. Often the measured data does not fit a rectangular pattern required by most of evaluation algorithms and export or import software modules are needed for data structure adaptation.

Figure 1.5. An example of inconvenient data preprocessing. Data reconstruction algorithm produces artificial structures on phase space.

It is also important to optimize, whenever it is possible, parameters of measurement system. Thus the pepper-pot quality of measurements is very sensitive to the device location and mask parameters.

A location point selection for multiple profiles emittance measurement.

Here is an illustration how the position of the measurement device could affect the accuracy of measurements. Figure 1.6 shows the principle of beam matrix determination by beam width measurements in three different locations. The waist location $X_{waist}$ and canonical beam matrix $S = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix}$ are completely described by the system of nonlinear equations where $S_{11}$ values are taken from the measurement and $a_i$ are defined by measurement locations.

$$
\begin{align*}
S_{11}^i &= \sqrt{S_{22}^i X_i^2 + S_{11}^i} ; \quad i = 1,2,3; \\
X_i &= a_i - X_{waist}
\end{align*}
$$

Figure 1.6. The principle of beam matrix determination by multiple profile measurements.

Measured data always includes some errors. Therefore a resulting beam matrix and correspondent emittance are considered only as a more or less successful approach to the true image. Figure 1.7 shows the emittance accuracy in presence of errors in the beam width measurements. It is assumed that all measurements were made with 1% rms uncertainty. As one can see the best results require a placement in the near to the waist. Otherwise this method of measurement degrades rapidly with distance.

Figure 1.7. The error analysis shows a rapid degradation of the multiple profile method with distance from waist point.
Accuracy and resolution

Another example for an error analysis is the investigation of the device resolution factor. Measurement quality or accuracy depends on numerous parameters and the resolution is an obvious factor which defines the precision of emittance measurements. Figure 1.8 shows the error areas for different phase space distributions. There are cases of Gaussian $u = e^{-R^2/a}$; quadratic $u = \begin{cases} 1 - \left(\frac{R}{b}\right)^2, & R < b \\ 0, & R \geq b \end{cases}$; and uniform $u = \begin{cases} 1, & R < c \\ 0, & R \geq c \end{cases}$ distributions. Generally, as one could expect, a smooth data means higher accuracy even using a lower resolution. For the Gaussian distribution this figure shows a very stable behaviour. Even few points of data are sufficient for reconstruction of the whole distribution.

![Figure 1.8](image)

Figure 1.8. Relative emittance error versus spatial resolution of the measurement system. The cases of Gaussian(1), quadratic(2) and uniform(3) charge distribution are shown.

2. DATA EVALUATION

Data Preprocessing

The original emittance data very often is noisy and includes missing and inconsistent data. Without proper preprocessing the low-quality data will lead to final low-quality results.

There is a number of data preprocessing techniques. Data cleaning can be applied to remove noise and correct inconsistencies in the data. Data integration merges data from multiple sources into a coherent data representation. Data transformation, such as normalization, geometrical distortion correction etc. may be applied. Data reduction can reduce the data size by aggregating, eliminating redundant features, or clustering, for instance. These techniques are not mutually exclusive and may work together. Data processing techniques, when applied before emittance evaluation, can substantially improve the overall quality of the result.

Data Cleaning

Missing values

There are following methods.

1. Ignore the tuple. For instance by assigning zero weight coefficients.
2. Fill in the missing value manually. In general, this approach is time consuming.
3. Use an average value over reasonable surrounding data.
4. Use the most probable value to fill in the missing value.

Quite often it happens that it is difficult to find an algorithmic rule for separation of the inconsistent data. In addition the human mind is still a flexible signal processing machine. Figure 2.1 shows an example of using the manually adjusted region of interest (ROI). In such case a researcher uses his own knowledge and experience which are not available by computer software.

![Figure 2.1](image)

Figure 2.1. Removing of non-consistent data. A definition of the region of interest could be saved and reused in series of similar measurements.

Noisy data

A noise component always exists in the experimental data. To get a better result one should take care about the signal-to-noise ratio already on the data acquisition stage by using proper low-noise detectors, signal matching, amplification range selection, integration time and input signal strength adjusting if possible.

![Figure 2.2](image)

Figure 2.2. The beamlet spot filtering. The original data is shown on the left, the filtered data with Gaussian filter with radiuses R=2 pixels - in the middle and R=5 pixels - on the right.

For effective noise reduction in the final phase space distribution one has to know the spectral properties of the noise and the signal itself. A de-noising process may
include several stages like applying of calibration coefficients and offsets, Poisson noise removing, spectral filtering etc.

**Figure 2.3.** Filter with large radius \( R \) reduces noise (right) but also changes a signal shape and statistical values.

Figure 2.2 shows filters with different strength applied to the beamlet spot from pepper-pot measurements. It works fine till the effective filter radius is much smaller than the characteristic dimension of spot details. Using filters is always a trade-off between noise reduction and signal shape distortion. Figure 2.3 shows the noise reduction and the error growth as a function of the two-dimensional filter effective radius for the data shown on the figure 2.2.

**Figure 2.4.** A wavelet-based beamlet spot de-noising shows a good result still preserving a spot shape.

Using wavelet decomposition for signal de-noising is another advanced technique which is successfully used for emittance data pre-evaluation. The left part of figure 2.4 shows a heavy noised pepper-pot beamlet spot. A result of de-noising based on 5-level Symlet decomposition [2] is shown on the right. In comparison with the Fourier method, the wavelet decomposition has a higher computation cost but gives a better result for non-periodic data.

**Data Integration and Transformation**

**Data Reduction**

Data reduction technique can be applied to obtain a reduced representation of the dataset that is much smaller in volume yet closely maintains the integrity of the original data. Strategies for the data reduction include the following:
- Attribute subset selection, where irrelevant, weakly irrelevant or redundant attributes or dimensions may be detected and removed.
- Dimensionality reduction, where encoding mechanisms are used to reduce the dataset size.
- Numerosity reduction, where the data are replaced or estimated by alternative, smaller data representation such as parametric models or nonparametric methods. The best example of this idea is a reduction of experimental data down to TWISS parameter set.

**Data deconvolution.**

It is very common that measured data is slightly or even significantly distorted by the measurement environment. This could happen for instance due to light scattering in viewing screens of pepper-pot devices, aberration in optics, inter-channel cross-talk, pulse jitter in longitudinal emittance measurements etc. In general the linear distortion is described by point-spread function (PSF) like it is shown on figure 2.5. One could try to correct distortions by applying a deconvolution filter to the experimental data.

**Figure 2.5.** An axial symmetric two-component point-spread function (PSF) reflects a pixels cross-talk (first exponent) and light scattering in viewing screen.

Figure 2.6 shows the original data and the result of shape correction for a beamlet spot in a pepper-pot measurements.

**Figure 2.6.** Correction of the light scattering in scintillator using Lucy-Richardson deconvolution algorithm gives in this example 30% of spot size reduction.

**Basic algorithms**

There are few traditional and well-established procedures for emittance calculation which could be considered as mutual standards. Those procedures are rather simple so one is sure that the same data will lead to the same result on any computer and with any kind of used software.

**Geometrical emittance**

The simplest way of emittance calculation is a 'measuring' of the area occupied by beam on the phase
plane. This could be done just by counting the elementary black pixels on figure 2.7.

\[ \varepsilon_{KV, level} = a = \frac{1}{\pi} \int_\Omega C \, dx_1 \, dx_2 = \frac{1}{\pi} \sum_{i \leq 1} \sum_{i \geq 1} C \ ; \]

\[ C = \begin{cases} 0, & U \leq a, \\ 1, & U > a; \end{cases} \]

A pixel selection criterion level \( a \) is necessary because otherwise the noise or low-weight peripheral data will lead to unreasonably growth of the emittance. Using a selection criterion makes this algorithm convergent when applying to Gaussian-like distributions with non-zero values on infinity. This level value is a subject of "expert" choice which makes this algorithm not completely self-sufficient.

![Figure 2.7](image)

Figure 2.7. Geometrical emittance calculation. The original data and selection mask are shown on the left and right correspondently.

The importance of expert skill is illustrated by figure 2.8. It shows three geometrical emittance curves for three canonical distributions having the same rms emittance. The argument on horizontal axis is a relative beam fraction taken into the calculation. There is a significant distinction between curves, so one should take a care when comparing emittances of differently shaped distributions.

![Figure 2.8](image)

Figure 2.8. The geometrical emittance as a function of beam fraction.

A fraction of 80% of a beam is a recommended value for the selection criterion. This is a region where all three curves on figure 2.8 are going close by.

### RMS emittance

The root-mean-square (rms) approach is another highly relevant method for compact description of the phase-space distribution. The rms values are calculated by averaging over whole data set and this makes this algorithm very stable for numerous kinds of charge distributions.

![Figure 2.9](image)

Figure 2.9. The rms approach replaces the original data with equivalent Gaussian distribution.

### Parametric data fit

As it was already mentioned, statistical methods show a good performance. The possibility to obtain a proper result in a single pass of calculation leads to excellent computation efficiency. Nevertheless sometimes the substantiations for using statistical methods are not strong enough. Thus the RMS method does not work well in presence of uniform background, non-symmetric or multicore data etc. The alternative is usage of parametric fitting of the data with Gaussian or more sophisticated functions. An example of data fitting by function with 13 parameters is shown on figure 2.10.

![Figure 2.10](image)

Figure 2.10. The parametric data fitting is a solution when the limited set of TWISS parameters is not sufficient for accurate representation of experimental data.

It should be mentioned that the computation cost of parametric fitting is much higher than the computation cost of rms methods. Also the computation time rises exponentially with the number of involved parameters. And, finally, gradient methods of optimisation are very
often not applicable and the random walk algorithm is only possible to get a result close to optimal.

**Optimal data fitting to the predefined acceptance**

When an accelerator or ion source is commissioned as a part of installation, the goal function for emittance optimization procedure may be defined in term of acceptance and capturing efficiency. Based on emittance invariance the algorithm tries to find a best possible configuration by matching an acceptance ellipse to measured data (see figure 2.11).

![Figure 2.11. A calculating of an achieved transmitting is a most convenient way to estimate the beam quality.](image)

During optimisation the algorithm may vary the acceptance ellipse position and shape by using constrains conditions defined by the existing environment.

### 3. SOFTWARE TOOLS

An emittance measurement is a very common kind of measurements in charged particles beam physics. It has a long history and well-established procedures for emittance evaluation. Those procedures could be a basis for building stand-alone software tools for standard data evaluation and for benchmarking of new methods. In the same time a wide spectrum of features of different measurements requires a fine tuning and individual algorithms nearly in every session of measurement. This argues for algorithmic tool collections allowing flexible combination of calculation and data presentation methods. For convenience such toolbox includes clear and completely transparent tools for performing of very basic operations on measured data. Also the calculation time of standard solutions is short enough to use the software for on-line analysis. All basic features of measured emittances are combined in a single output image as it is shown on the figure 3.1.

The possibility of a sophisticated analysis of complicated data is the main advantage of the software organized in form of a toolbox. All pre-evaluation problems shown above were processed as an extension subset of standard toolbox procedures [2].

![Figure 3.2. Some ways to present a phase space distribution. Flat graphs give a good common overview of whole distribution. Small details like ripple or noise easy to see on 3-D pictures.](image)

![Figure 3.1. A typical output of standard data evaluation algorithm includes a phase picture, calculated TWISS parameters, fraction analysis curve etc.](image)

### Data presentation

One of the aims of the emittance evaluation is a better understanding of properties of the investigated phenomena. Also it is important to provide a knowledge, or expressions exchange between peoples or groups of peoples. The proper presentation of results makes this task more transparent and effective.

Figure 3.1 shows some possibilities for phase space visualization. In addition to the standard output one could find a reason to represent a data as a contour, 3D plot or in some sophisticated way (figure 3.3).
CONCLUSIONS

An emittance measurement technique has a history almost as long as a history of charged particles beams. Requirements to measurements quality are defined by demanded quality factor of state-of-art accelerated beams and the progress in measurements determined by progress in electronics and particle detectors. Starting with slow successive slit-wire method a performance increased significantly with modern CCD detectors with few millions pixels of data per single shoot. The variety devices and methods are covered with proper algorithms of data evaluation. From the other hand there is still a lot of possibilities for algorithm improvements or adaptation of numerous sophisticated ideas from closely related fields like radars or astronomy.

REFERENCES